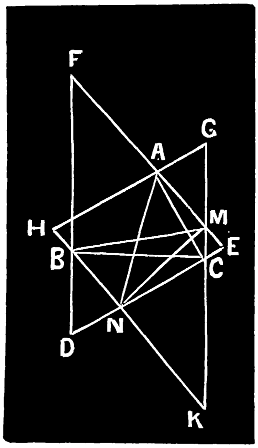
II. Solution by PROPOSER.

Let ABC be the given triangle area $\triangle$; DEF, GHK the circumscribed triangles; FE, GK intersecting in M; DC, HK intersecting in N. Join BM, AN, MN. The triangles DEF, GHK are equiangular. $\angle BAM=\angle BCM=\angle ACN=\angle ABN=$ a right angle.



$\therefore$ A circle passes through A, M, C, B and A, C, N, B, respectively.

$\therefore \angle AMB=\angle ACB=\angle ANB$.

$\therefore AM=BN$ and is parallel to it, $MN=AB$ and is parallel to it.

$\therefore AF=NK$, $ME=HB$, $\therefore FE=HK$ and $DEF=GHK$. Let A=area of DEF, then $2A=EC.AC+FA.AB+DB.BC+\triangle=b^2\cot A+c^2\cot B+a^2\cot C+\triangle=c^2\cot A+a^2\cot B+b^2\cot C+\triangle$.

$\therefore 4A=[a^4+b^4+c^4-bc(b^2+c^2)-ac(c^2+a^2)-ab(a^2+b^2)+4\triangle^2]/2\triangle$.

Also solved by F. D. Posey, San Mateo, California; L. E. Newcomb, Los Gatos, California; and J. Scheffer, Hagerstown, Md.

MECHANICS.

165. Proposed by O. W. ANTHONY, DeWitt Clinton High School, New York City.

Find the approximate form of a tower of circular cross section 1000 feet high and having a radius of lower base 20 feet, and so constructed that all the parts of the structure shall be subject to the same stress, due to the weight of the part of the tower above.

III. Solution by CHRISTIAN HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O.

Let w=the weight per unit of volume; s=the stress (pressure per square inch); y=the radius of cross section at distance x from top.

Then πy^2=the area of cross section at distance x from top, and, by the conditions of the problem, the increment of the area for any increment of x multiplied by s, must equal the increment of the weight, we get, in the limit, $\frac{dy}{dx}=\frac{w}{2s}y$. Integrating, we find $y=ce^{(w/2s)x}$. Now since $y=20$ when $y=1000$, $c=20e^{-(1000w/2s)}$.

$\therefore y=20e^{(w/2s)(x-1000)}$, which, by assigning proper values to w and s, will give the form of an element of the lateral surface of the tower. If we call $w=\frac{1}{10}$ and $s=20$, the radii of the tower at different elevations would be as follows: At 100 feet, 12.13 feet; at 200, 7.36; at 300, 4.46; at 400, 2.70; at 500, 1.64; at 600, .996; at 700, .604; ot 900, .222; at 990, .1417; and at top, .1348 feet or 1.6 inch.

This result will fulfill the required condition as to stress everywhere except near the top.

166. Proposed by G. B. M. ZERR, A. M., Ph D., Parsons, W. Va.

If a gravitating particle of mass m be placed at the point (a, b, c) prove